Exchange

M. Utku Ünver
"Micro Theory"

Boston College

General Equilibrium

So far we have been analyzing the behavior of a single consumer. In this chapter, we will see how consumers interact in a **market** setting and how that affects the prices.

This kind of analysis is called **General Equilibrium Analysis**.

Suppose there are two consumers with their respective endowments. They meet at a market and trade some of their goods. How do we determine the relative **prices** of the goods, and the consumers' choice bundles?

An Exchange Economy

Consumers: A and B
Goods: 1 and 2
Endowments: $\omega = (\omega^A, \omega^B) = ((\omega_1^A, \omega_2^A), (\omega_1^B, \omega_2^B))$
Total endowment of goods in the economy:

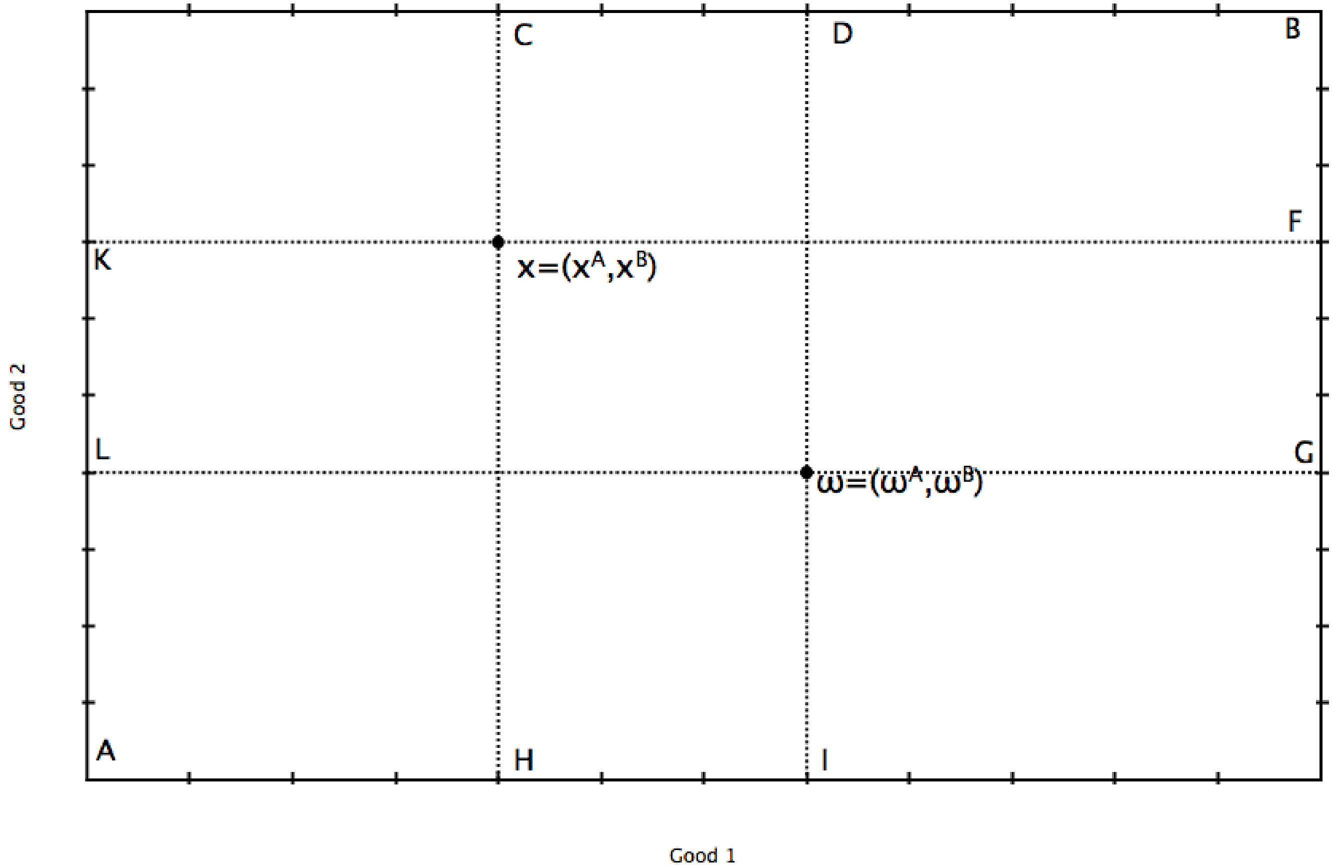
- Good 1: $\omega_1^A + \omega_1^B$
- Good 2: $\omega_2^A + \omega_2^B$

Demands: $X = (x^A, x^B) = ((x_1^A, x_2^A), (x_1^B, x_2^B))$
Demands should be feasible:

- Good 1: $x_1^A + x_1^B = \omega_1^A + \omega_1^B$
- Good 2: $x_2^A + x_2^B = \omega_2^A + \omega_2^B$

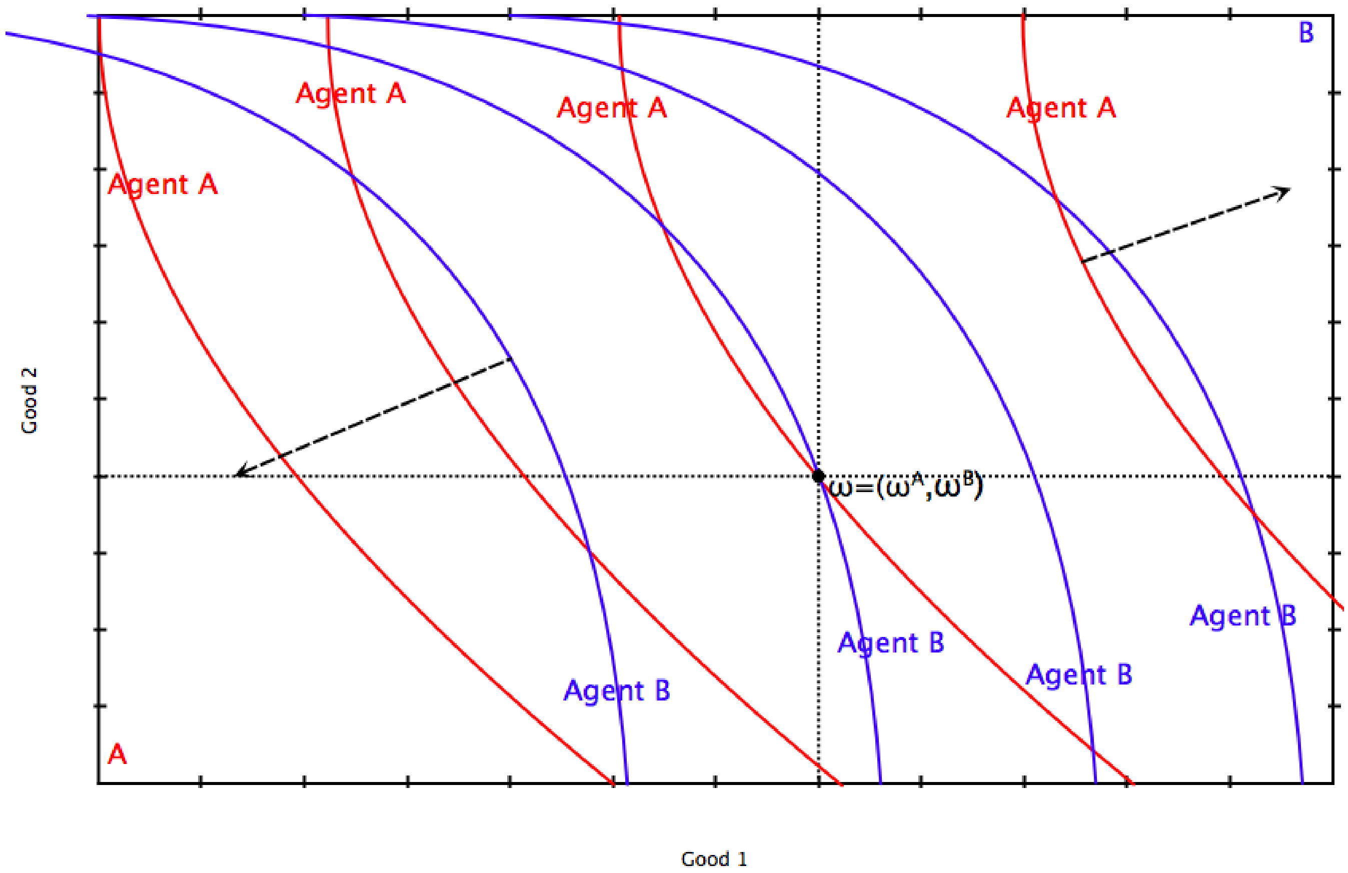
Edgeworth Box

For equilibrium analysis we use a useful tool called **Edgeworth Box**



ω is endowment: $(\overline{AI},\overline{AL})$ is Agent A's $(\overline{BD},\overline{BG})$ is Agent B's.
X is consumption: $(\overline{AH},\overline{AK})$ is Agent A's, $(\overline{BC},\overline{BF})$ is Agent B's.
Agent A sells $\overline{HI}$ units of good 1 and buys $\overline{KL}$ units of good 2.

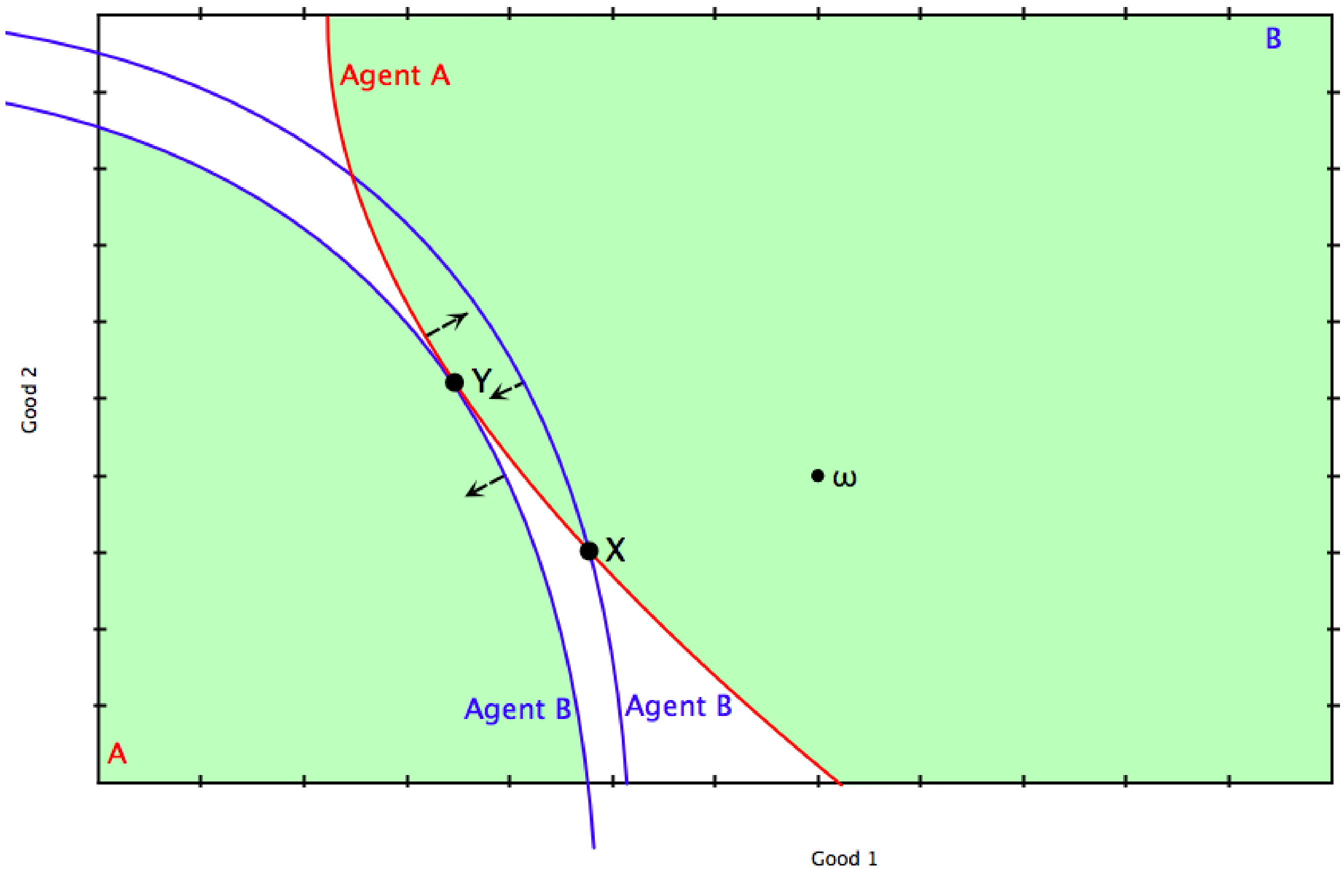
The following figure shows the preferences of two agents in an Edgeworth box.

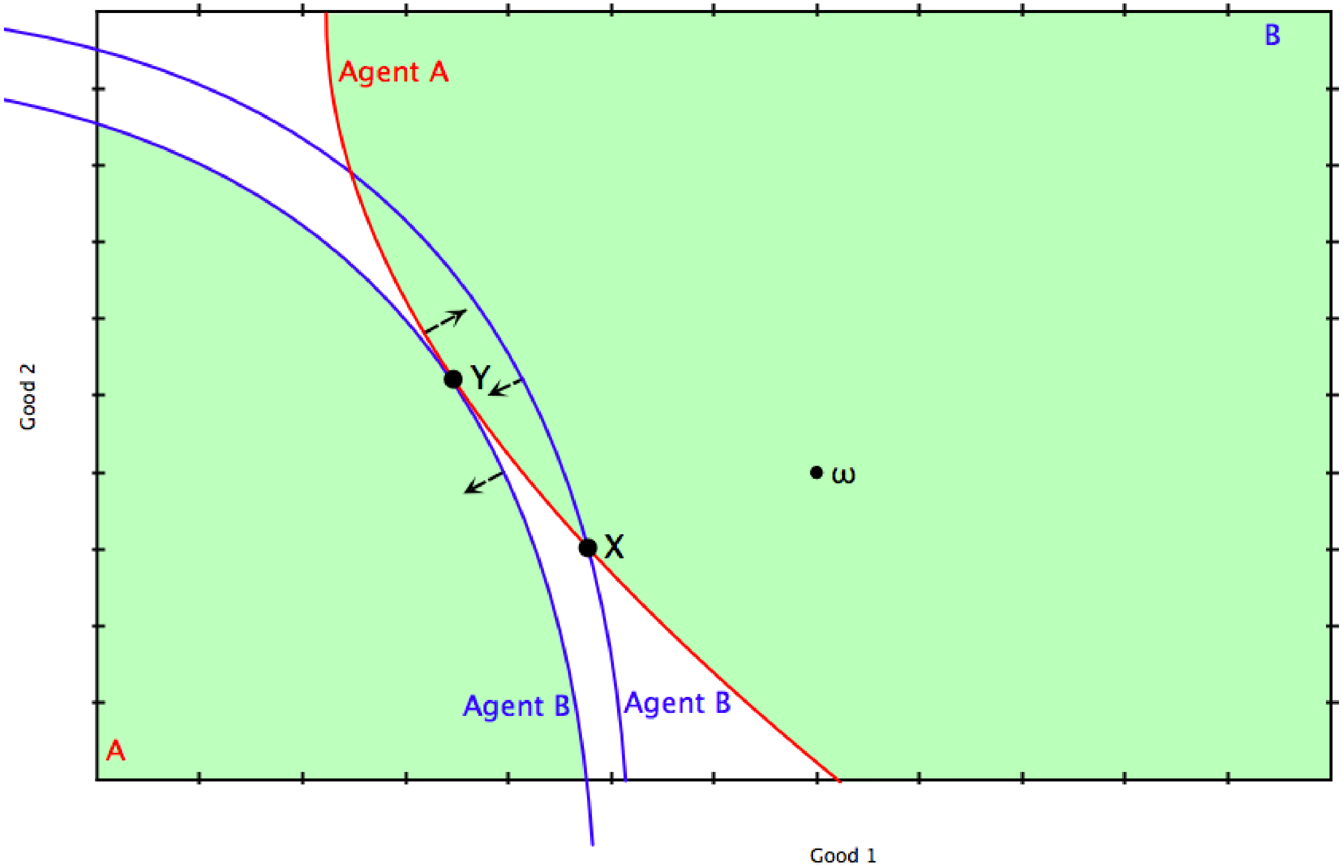


Pareto efficiency

If we can make some agents better off without making any agent worse off, then an allocation is not Pareto efficient.

Formally, an allocation is **Pareto efficient** if it is not possible that we can make some agents better off without making any agent worse off.





Allocation Y is Pareto efficient. We cannot make either agent better off than she is at Y, without making the other worse off.

Allocation X is not Pareto efficient. Moreover, allocation Y Pareto dominates allocation X, since it gives more utility to agent B and gives the same utility to agent A. That is, it makes agent B better off without making A worse off.

Example: Suppose $U^A = (x_1^A)(x_2^A)$ and $U^B = (x_1^B)(x_2^B)^2$. The initial endowments are given by $\omega^A = (1, 1)$ and $\omega^B = (2, 1)$. Find the set of Pareto efficient allocations (also known as the **contract curve**).

Solution:

1. We need to equalize the MRS of agent A and MRS of agent B for Pareto efficiency.
2. For the allocation to be feasible we need
$x_1^A + x_1^B = \omega_1^A + \omega_1^B = 1 + 2 = 3$ and
$x_2^A + x_2^B = \omega_2^A + \omega_2^B = 1 + 1 = 2.$
Then $x_1^B = 3 - x_1^A$ and $x_2^B = 2 - x_2^A$

$$MRS^A = MRS^B \Rightarrow$$

$$-\frac{\partial U^A / \partial x_1^A}{\partial U^A / \partial x_2^A} = -\frac{\partial U^B / \partial x_1^B}{\partial U^B / \partial x_2^B} \Rightarrow$$

$$-\frac{x_2^A}{x_1^A} = -\frac{x_2^B}{2x_1^B} \Rightarrow$$

Use the feasibility condition we found above

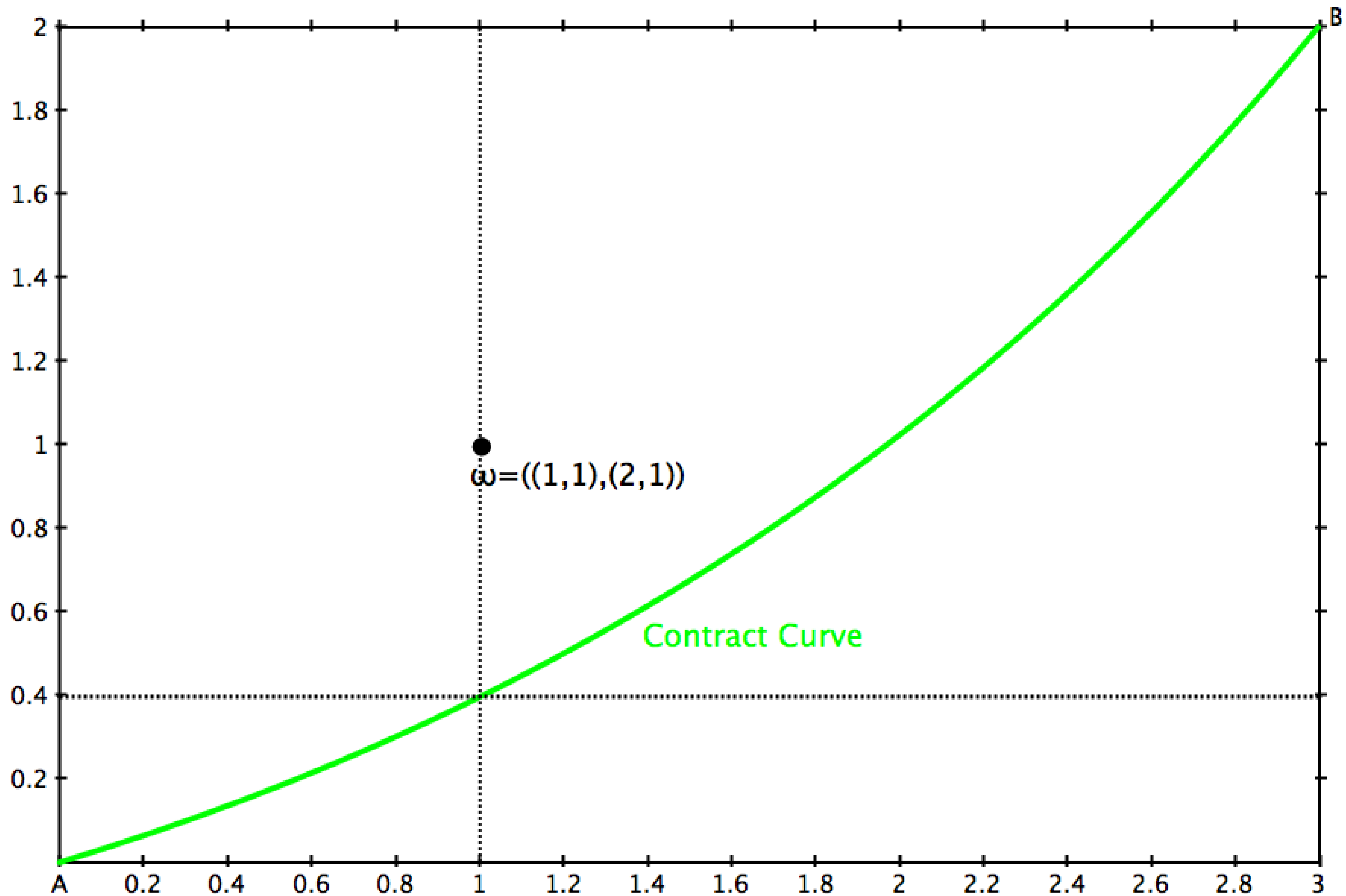
to isolate one person's choice, for example A's

$$-\frac{x_2^A}{x_1^A} = -\frac{2 - x_2^A}{2(3 - x_1^A)} \Rightarrow$$

$$6x_2^A - 2x_1^A x_2^A = 2x_1^A - x_1^A x_2^A \Rightarrow$$

$$\boxed{x_2^A = \frac{2x_1^A}{6 - x_1^A}}$$

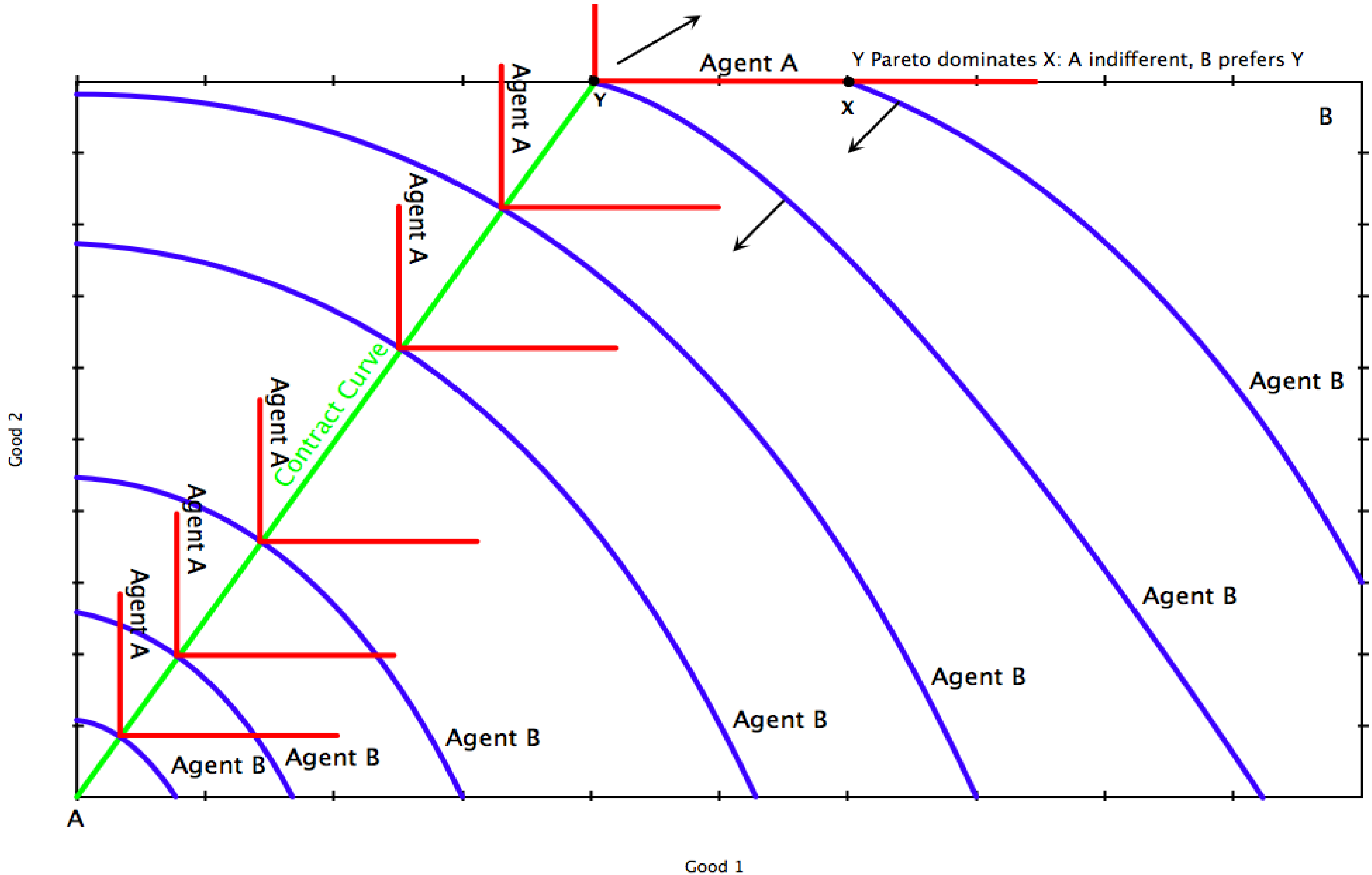
The plot of this above equation (the contract curve:)



For example, if $x_1^A = 1$ then $x_2^A = \frac{2}{5}$ and, by feasibility, $x_1^B = 2$ and $x_2^B = \frac{8}{5}$.

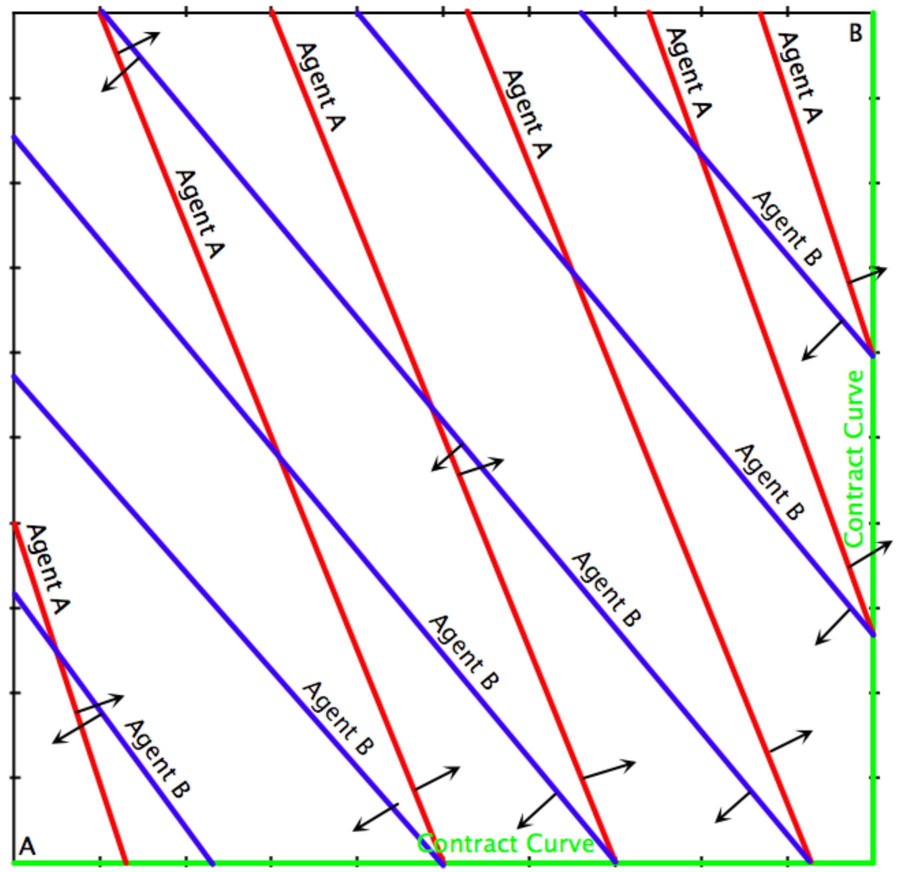
What if there are kinks at preferences?

Example: A has a perfect-complement utility function, while B has some smooth strictly convex preferences.



What if there are corner solutions?

Example: If both of the agents have perfect-substitutes preferences with different MRS, we will have the set of Pareto efficient allocations as the edges of the Edgeworth box as shown as lighter mirrored L-shaped curve.



Note that if both of the agents have the same MRS in this case, all the points in the Edgeworth box are Pareto efficient. Why?

Example: $U^A = (x_1^A)(x_2^A)$ and $U^B = x_1^B + 2x_2^B$. The initial endowments are given as $\omega^A = (1,1)$ and $\omega^B = (1,1)$. Find the set of Pareto efficient allocations.

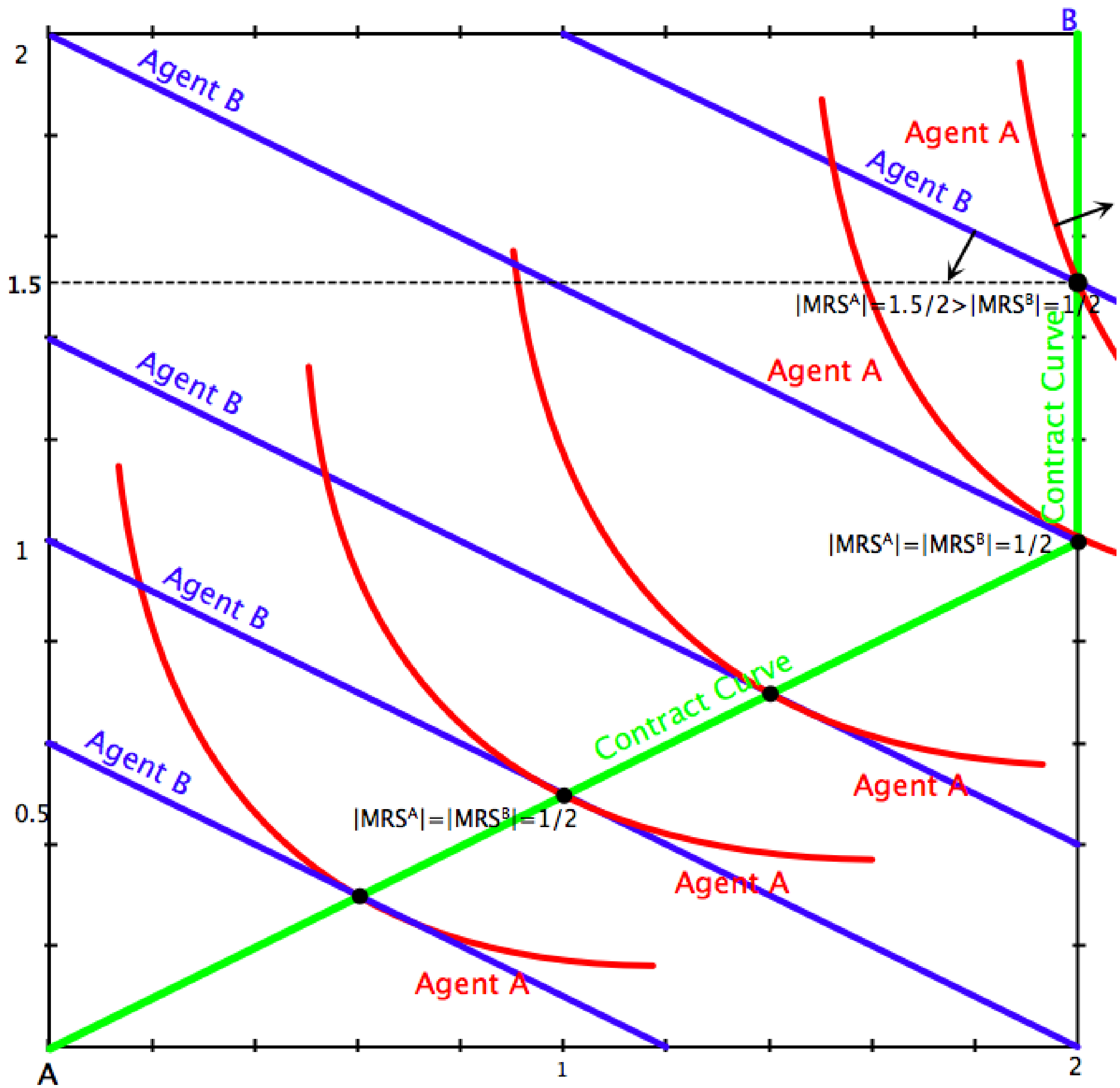
Solution:

$$MRS^A = MRS^B \implies$$

$$-\frac{\partial U^A/\partial x_1^A}{\partial U^A/\partial x_2^A} = -\frac{\partial U^B/\partial x_1^B}{\partial U^B/\partial x_2^B} \implies$$

$$-\frac{x_2^A}{x_1^A} = -\frac{1}{2} \implies \boxed{x_2^A = \frac{1}{2}x_1^A}$$

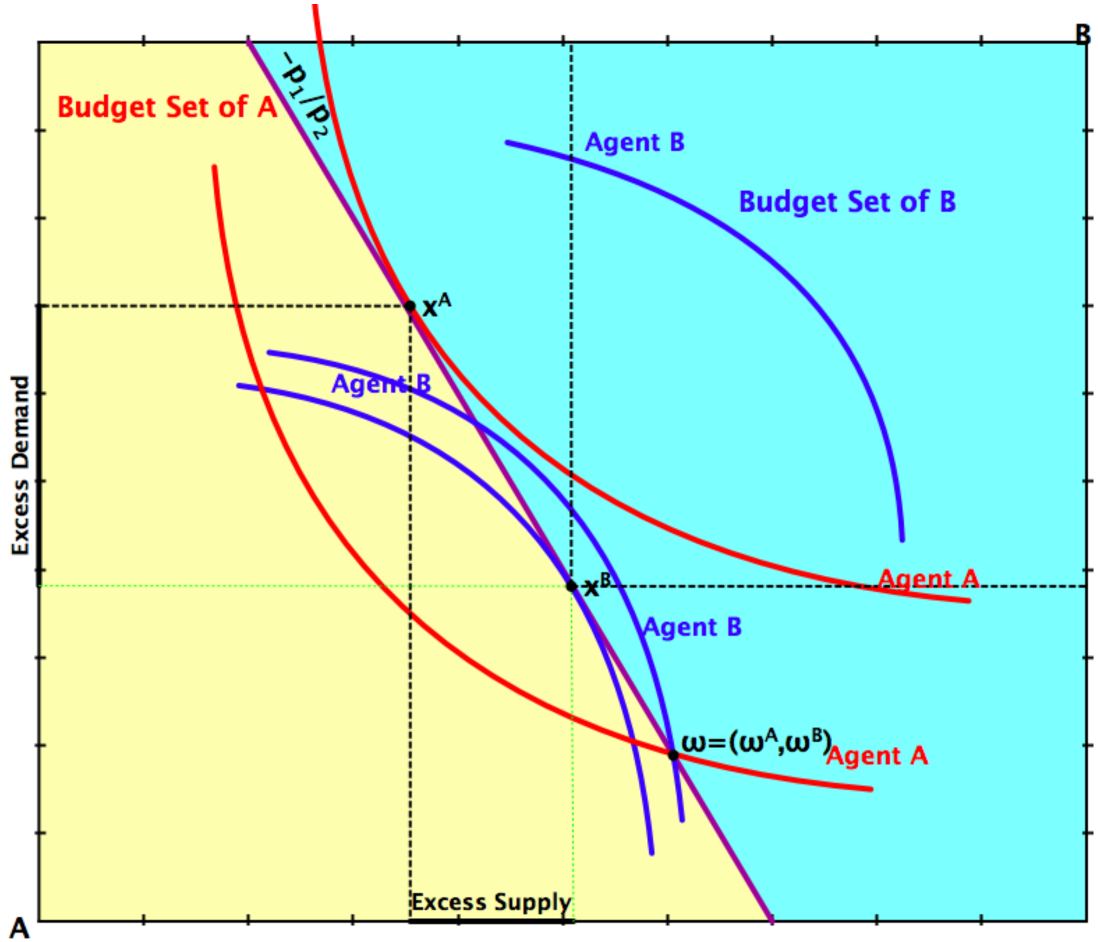
The plot of above equation (the contract curve:) Observe that there are parts of contract curve where MRS's are not equal, corner solutions.



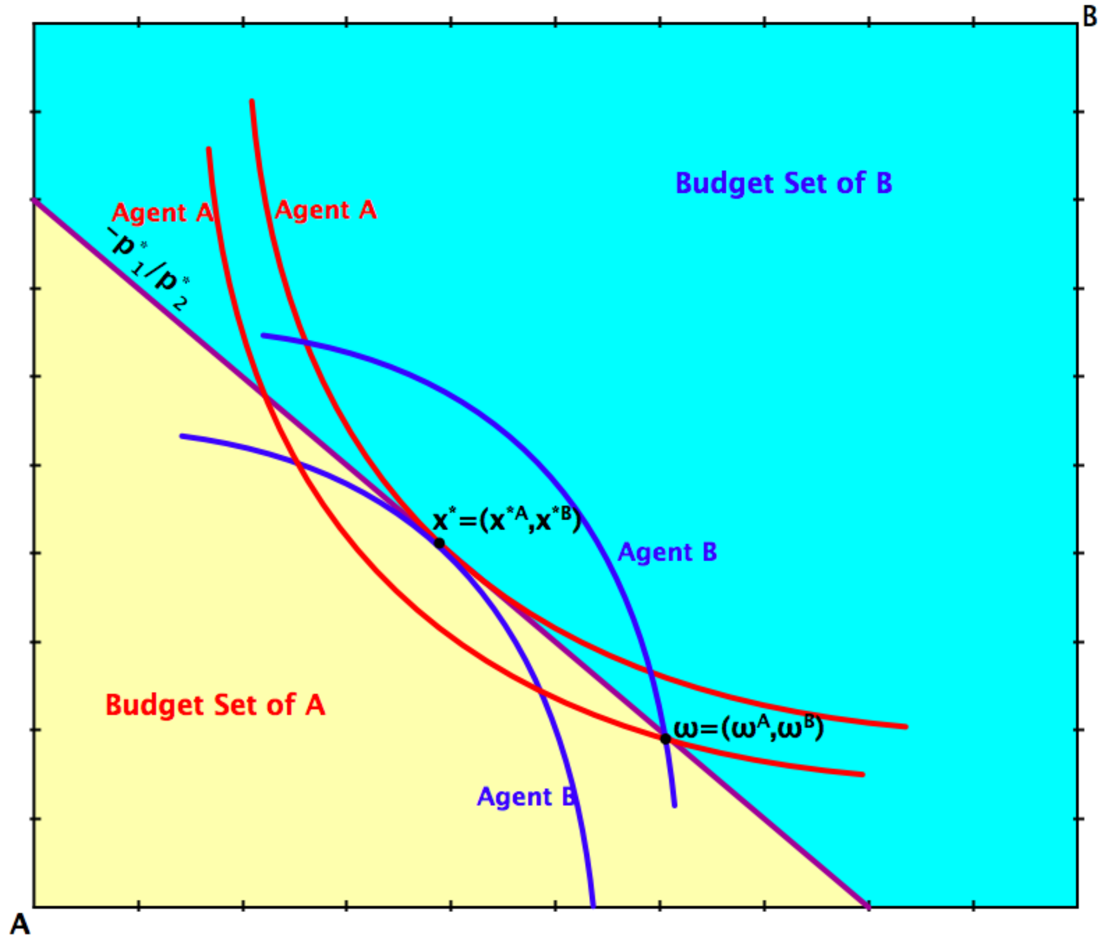
Competitive Equilibrium

A price vector (p_1, p_2) and an allocation $(X^A, X^B) = ((x_1^A, x_2^A), (x_1^B, x_2^B))$ is a **competitive equilibrium** if

1. each person is choosing the most preferred bundle in his budget set and
2. there is neither excess demand nor excess supply for any good. (i.e., markets clear)



In the above figure, the price ratio p_1/p_2 specified by slope of the budget line(s) and the allocation specified by (x^A, x^B) is not a competitive equilibrium. While each agent is maximizing their utilities, the markets do not clear. There is excess demand for good 2 and excess supply for good 1.



The tangency point in the above Edgeworth box figure is the competitive equilibrium for that economy. The price ratio p_1^*/p_2^* together with the allocation $x^* = (x^{*A}, x^{*B})$ is a competitive equilibrium for this economy.

Example: Suppose $U^A = (x_1^A)(x_2^A)$ and $U^B = (x_1^B)(x_2^B)^2$. The endowments are given by $\omega^A = (1, 1)$ and $\omega^B = (2, 1)$. Find the competitive equilibrium in this economy.

Solution:

Step 1 : First we find the demand functions of the agents for both goods. Let $p_1 = 1$ (numeraire good,) and $p_2 = p$ (unknown, we can only determine one of the prices)

The demand functions of agent A are as follows (Cobb-Douglas preferences):

$x_1^A = \frac{1}{2}\frac{m^A}{p_1} = \frac{1}{2}\frac{p_1+p_2}{p_1} = \frac{1}{2}(1+p)$ where m^A =the value of the endowment of agent A.

$x_2^A = \frac{1}{2}\frac{m^A}{p_2} = \frac{1}{2}\frac{1+p}{p}$.

The demand functions of agent B are as follows (Cobb-Douglas preferences):

$x_1^B = \frac{1}{3}\frac{m^B}{p_1} = \frac{1}{3}(2+p)$ and $x_2^B = \frac{2}{3}\frac{m^B}{p_2} = \frac{2}{3}\frac{2+p}{p}$ where $m^B =$ the value of the endowment of agent B.

Step 2 : Clearing the markets.

$$x_1^A + x_1^B = \omega_1^A + \omega_1^B = 3 \implies$$
$$\frac{1}{2}(1+p) + \frac{1}{3}(2+p) = 3 \implies$$
$$p = \frac{11}{5}.$$

Then we can find the equilibrium allocation using the demand functions: $x_1^A = \frac{8}{5}, x_2^A = \frac{8}{11}, x_1^B = \frac{7}{5}$ and $x_2^B = \frac{14}{11}$.

Therefore, the competitive equilibrium is $(1, \frac{11}{5})$ =competitive price and $((\frac{8}{5}, \frac{8}{11}), (\frac{7}{5}, \frac{14}{11}))$ =competitive allocation.

Example: Suppose $U^A = (x_1^A)(x_2^A)^2$ and $U^B = \min\{x_1^B, x_2^B\}$. The initial allocations are given as $\omega^A = (0, 2)$ and $\omega^B = (2, 0)$. Find the competitive equilibrium in this economy.

Walras' Law

(Walras' Law)

Suppose there are k goods in the exchange economy. If (k-1) markets clear, then the k^{th} market clears as well.

Welfare Economics

Theorem (First Fundamental Theorem of Welfare Economics)

Any competitive equilibrium allocation is Pareto efficient.

Theorem (Second Fundamental Theorem of Welfare Economics)

Supoose that preferences are convex. Then any interior Pareto efficient allocation can be obtained as a competitive equilibrium allocation from some initial endowment.